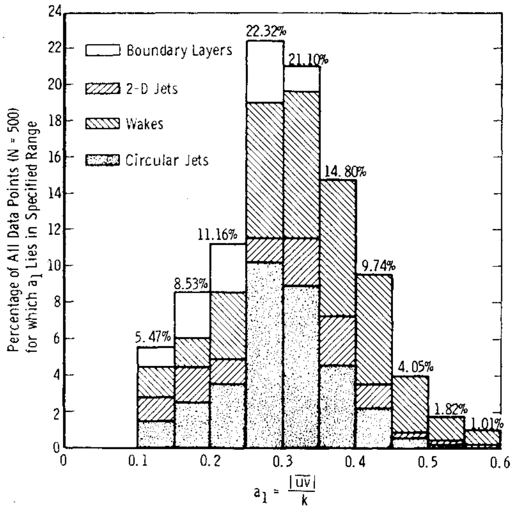

Fig. 3 Distribution of the observed values of the parameter a_1 for the data surveyed.

layers and two-dimensional jets) have not been separately plotted in this note. It can be seen that 70% of the data fall in the range $0.2 < a_1 < 0.4$, and that $a_1 = 0.3$ is a reasonably good value judging from the approximately 500 data points examined. In regard to this correlation it should be pointed out that the parameter a_1 must approach zero at the centerline of a free mixing flow, since the shear stress at the centerline must be zero by reason of flow symmetry, while experimental evidence indicates that the turbulent kinetic energy remains nonzero. The value of the parameter a_1 then must increase from zero to its nominal value over a portion of the mixing region. The experiments of Sami[9] are sufficiently detailed for this variation to be investigated, and this limited evidence indicates that the portion of the mixing region over which this variation occurs is small. A similar variation is also seen to occur in boundary-layer flows (see Bradshaw[5]). This variation in the value of the parameter a_1 may explain the slight biasing toward the smaller values of a_1 apparent in Fig. 3.

Conclusions

Based on a study of a substantial amount of turbulent shear stress and kinetic energy data, the existence of a linear relationship between turbulent shear stress and kinetic energy is reasonably well supported over a wide range of experimental conditions in incompressible flow. In the regions of flow where a constant ratio between turbulent shear and turbulent kinetic energy can be expected to exist, a reasonable value for the constant of proportionality may be taken to be 0.3. There is not yet sufficient evidence available for the variation of this constant of proportionality to be modelled in other flow regions, such as those in which the turbulent shear stress approaches zero while the turbulent kinetic energy does not.

References

[1] Prandtl, L., "Bericht uber Untersuchungen zur Ausgebildenten Turbulenz," *Zeitschrift fur angewandte Mathematik and Mechanik*, Vol. 5, No. 2, April 1925, pp. 136–139.

[2] Dryden, H. L., *Advances in Applied Mechanics*, Vol. 1, Academic Press, New York, 1948, pp. 1–40.

[3] Bradshaw, P., Ferriss, D. H., and Atwell, N. P., "Calculation of Boundary Layer Development Using the Turbulent Energy Equation," *Journal of Fluid Mechanics*, Vol. 28, Pt. 3, May 1967, pp. 593–616.

[4] Lee, S. C. and Harsha, P. T., "The Use of Turbulent Kinetic Energy in Free Mixing Studies," AIAA Paper 69-683, San Francisco, Calif., 1969; also *AIAA Journal*, to be published.

[5] Bradshaw, P., "The Turbulence Structure of Equilibrium Boundary Layers," *Journal of Fluid Mechanics*, Vol. 29, Pt. 4, Sept. 1967, pp. 625–645.

[6] Townsend, A. A., "The Fully Developed Turbulent Wake of a Circular Cylinder," *Australian Journal of Scientific Research*, Ser. A, Vol. 2, Dec. 1949, pp. 451–468.

[7] Lee, S. C., "A Study of the Two-Dimensional Free Turbulent Mixing between Converging Streams with Initial Boundary Layers," Ph.D dissertation, 1966, Univ. of Washington, Seattle, Wash.

[8] Carmody, T., "Establishment of the Wake Behind a Disk," *Transactions of the ASME, Series D: Journal of Basic Engineering*, Vol. 86, Dec. 1964, pp. 869–882.

[9] Sami, S., "Velocity and Pressure Fields of a Diffusing Jet," Ph.D dissertation, 1966, Univ. of Iowa, Iowa City; also Sami, S., Carmody, T., and Rouse, H., "Jet Diffusion in the Region of Flow Establishment," *Journal of Fluid Mechanics*, Vol. 27, Pt. 2, Feb. 1967, pp. 231–252.

[10] Bradshaw, P., Ferriss, D. H., and Johnson, R. F., "Turbulence in the Noise-Producing Region of a Circular Jet," *Journal of Fluid Mechanics*, Vol. 19, Pt. 4, August 1964, pp. 591–625.

[11] Gibson, M. M., "Spectra of Turbulence in a Round Jet," *Journal of Fluid Mechanics*, Vol. 15, Pt. 2, Feb. 1963, pp. 161–173.

[12] Curtet, R. and Ricou, F. P., "On the Tendency to Self-Preservation in Axisymmetric Ducted Jets," *Transactions of the ASME, Ser. D, Journal of Basic Engineering*, Vol. 86, Dec. 1964, pp. 765–776.

[13] Zawacki, T. S. and Weinstein, H., "Experimental Investigation of Turbulence in the Mixing Region between Coaxial Streams," CR-959, Feb. 1968, NASA.

Exponential Kernel Approximation in Radiative Energy Transfer within a Hydrogen Plasma

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Nomenclature

e_ω = Planck's function
E_2 = second exponential integral function
F_2 = transmission function, Eq. (1), cm^{-1}atm^{-1}
L = plate spacing, cm
Ne = electron density, cm^{-3}
P = total pressure, atm
P_H = partial pressure of atomic hydrogen, atm
Q = heat source per unit volume
q_R = radiative heat flux
T = temperature, °K
T_1 = boundary temperature, °K
T_c = centerline temperature, °K
u = pressure path length, $u = P_H y$, cm-atm
u_o = total pressure path length, $u_o = P_H L$, cm-atm
y = distance measured from lower boundary
κ_ω = spectral absorption coefficient, cm^{-1}
λ = thermal conductivity
σ = Stefan-Boltzmann constant
ω = wave number, cm^{-1}

Introduction

THE object of this Note is to investigate the influence of the exponential kernel approximation on radiative energy transfer within high-temperature gases. In order to

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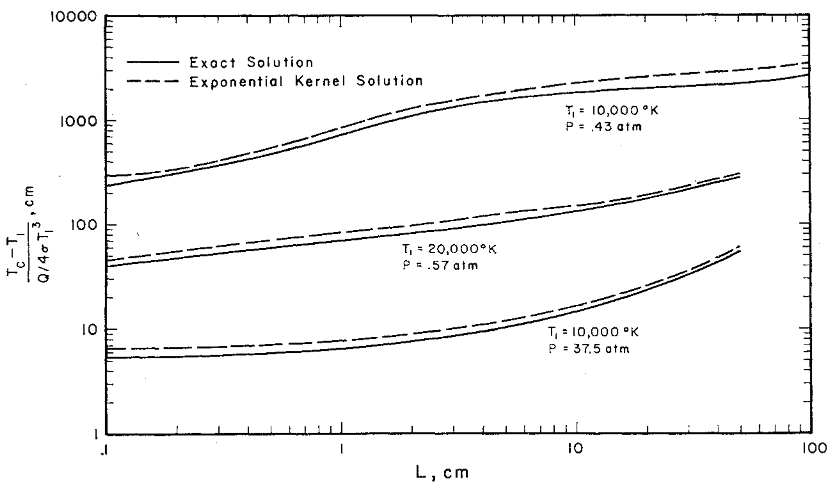

Fig. 1 Centerline temperature results with conduction neglected.

illustrate this influence, results are presented for a hydrogen plasma.

The energy equation for radiative transfer involves integral or integro-differential equations. If a gray gas is assumed, the kernel function for the integrals is the exponential integral function $E_2(t)$. The kernel function for real (nongray) gases is a complicated function of the particular gas and conditions under consideration.

If the exponential approximation is made; that is, replacing $E_2(t)$ by an exponential, then the nongray kernel functions can be expressed in terms of the spatial derivative of either the emissivity[1] or, for linearized radiation, the modified emissivity.[2] The exponential approximation has been used to study infrared radiation[3,4] and also plasma radiation.[5,6] In infrared gaseous radiation, the results obtained by using the exponential approximation are in close agreement with exact results.[4] The linearized radiative flux can be formulated without making the exponential approximation.[7] This formulation is in terms of transmission functions.

In this Note transmission functions are presented for a hydrogen plasma at temperatures of 10,000°K and 20,000°K and for electron densities of 10^{16} and 10^{17} cm^{-3}. These functions are used to study radiation in a hydrogen plasma bounded by two parallel black boundaries and within which there is a uniform heat source. These results are compared to earlier results which contained the exponential approximation.[5] Linearized radiation and local thermodynamic equilibrium are assumed.

Analysis

For nongray linearized radiation, q_R has been given in terms of transmission functions[7]

$$F_2(u) = \frac{1}{4\sigma T_1^3} \int_0^\infty \frac{\kappa_\omega}{P_H} \left(\frac{\partial e_\omega}{\partial T}\right)_{T_1} E_2\left(\frac{\kappa_\omega}{P_H} u\right) d\omega \quad (1)$$

For boundaries at the temperature T_1, the linearized radiative flux becomes

$$q_R = 8\sigma T_1^3 \int_0^u [T(u') - T_1] F_2(u - u') du' - 8\sigma T_1^3 \int_u^{u_0} [T(u') - T_1] F_2(u' - u) du' \quad (2)$$

where the linearization has been taken about T_1, and κ_ω thus corresponds to this temperature. In order to calculate $F_2(u)$ it is necessary to have the spectral absorption coefficient κ_ω as a function of wave number, temperature and density. The calculations for F_2 reported in this paper were made by using the methods for calculating the spectral absorption coefficient discussed in Ref. 8.

For the problem of uniform heat generation, and with conduction neglected, the equation for conservation of energy is

$$\frac{u}{u_0} - \frac{1}{2} = \frac{4}{3} \int_0^u \theta(u') F_2(u - u') du' - \frac{4}{3} \int_u^{u_0} \theta(u') F_2(u' - u) du' \quad (3)$$

where $\theta = 6\sigma T_1^3 (T - T_1)/LQ$. When conduction within the gas is included, the energy equation becomes

$$u_0 \frac{d\phi}{du} + \frac{u}{u_0} - \frac{1}{2} = \left(\frac{8\sigma T_1^3 L}{\lambda}\right) \left\{ \int_0^u \phi(u') F_2(u - u') du' - \int_u^{u_0} \phi(u') F_2(u' - u) du' \right\} \quad (4)$$

where $\phi = \lambda(T - T_1)/QL^2$ and the boundary condition is $\phi(0) = 0$.

Results

The results obtained for $F_2(u)$ are given in Table 1. Equations (3) and (4) were solved numerically by the method of undetermined parameters,[9] in the same manner as described in Ref. 5. The results are shown in Figs. 1 and 2 along

Table 1 Transmission functions

	$F_2(u)$, cm^{-1} atm^{-1}		
u, cm-atm	T_1 = 10,000°K $Ne = 10^{16}$ cm^{-3}	T_1 = 10,000°K $Ne = 10^{17}$ cm^{-3}	T_1 = 20,000°K $Ne = 10^{17}$ cm^{-3}
0	0.246	0.214	1.09
5×10^{-5}	5.01×10^{-2}	9.78×10^{-2}	0.794
1×10^{-4}	3.57×10^{-2}	7.33×10^{-2}	0.687
5×10^{-4}	1.35×10^{-2}	3.45×10^{-2}	0.461
1×10^{-3}	8.66×10^{-3}	2.33×10^{-2}	0.385
5×10^{-3}	3.82×10^{-3}	8.61×10^{-3}	0.248
1×10^{-2}	2.98×10^{-3}	5.81×10^{-3}	0.206
5×10^{-2}	1.69×10^{-3}	2.91×10^{-3}	0.134
1×10^{-1}	1.19×10^{-3}	2.25×10^{-3}	0.107
5×10^{-1}	4.37×10^{-4}	1.43×10^{-3}	4.22×10^{-2}
1	3.59×10^{-4}	1.34×10^{-3}	2.16×10^{-2}
5	2.94×10^{-4}	1.21×10^{-3}	6.83×10^{-3}
10	2.73×10^{-4}	1.12×10^{-3}	5.29×10^{-3}
50	2.24×10^{-4}	8.06×10^{-4}	1.98×10^{-3}
1×10^{2}	2.01×10^{-4}	6.06×10^{-4}	1.02×10^{-3}
5×10^{2}	1.31×10^{-4}	1.66×10^{-4}	1.12×10^{-4}
1×10^{3}	9.35×10^{-5}	7.66×10^{-5}	2.74×10^{-5}
2.5×10^{3}	4.79×10^{-5}	2.84×10^{-5}	1.74×10^{-6}
5×10^{3}	2.40×10^{-5}	1.40×10^{-5}	6.06×10^{-8}

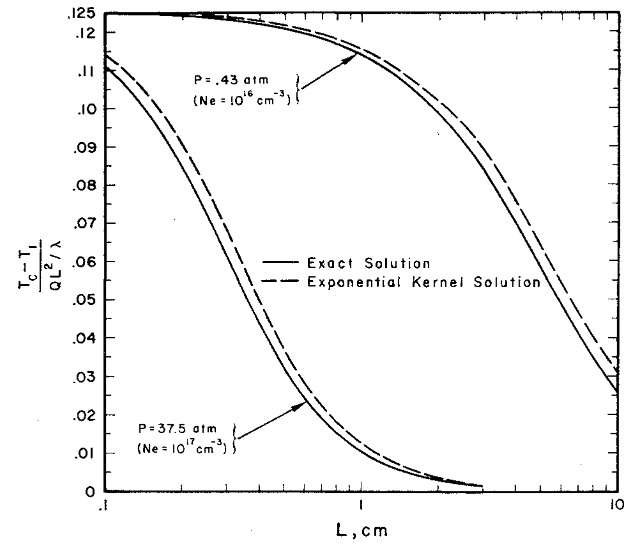

Fig. 2 Centerline temperature results with conduction included, T_1 = 10,000°K.

with the results obtained by making the exponential approximation.[5]

The results for conduction neglected are shown in Fig. 1. In order to see the influence of the exponential approximation, the results for a typical plate spacing can be examined. For a plate spacing of $L = 1$ cm., the exact solution differs from the exponential kernel solution by approximately 20% for $T_1 = 10{,}000°K$ and $P = 0.43$ atm; by 15% for $T_1 = 10{,}000°K$ and $P = 37.5$ atm; and by 22% for $T_1 = 20{,}000°K$ and $P = 0.57$ atm.

Figure 2 shows the results for $T_1 = 10{,}000°K$ with conduction included. In order to see the influence of the exponential approximation in this case, the results can be examined when radiation and conduction are equally important mechanisms of heat transfer. This occurs when the centerline temperature is reduced to one-half its maximum value, which is the pure conduction value,[5] 0.125. For $T_1 = 10{,}000°K$ and $P = 0.43$ atm, the exact dimensionless centerline temperature is reduced by one-half at $L = 4.6$ cm., and the per cent difference between the exact and exponential kernel solutions is about 10%; for $T_1 = 10{,}000°K$ and $P = 37.5$ atm, the difference is 14% at $L = 0.29$ cm. For $T_1 = 20{,}000°K$, the results are similar to the aforementioned results. Thus when conduction is included, the exponential kernel approximation has a relatively small influence on the total heat-transfer problem.

References

[1] Penner, S. S., *Quantitative Molecular Spectroscopy and Gas Emissivities*, Addison-Wesley, Reading, Mass., 1959, p. 380.

[2] Gille, J. and Goody, R. M., "Convection in a Radiating Gas," *Journal of Fluid Mechanics*, Vol. 20, 1964, pp. 47–79.

[3] Cess, R. D., Mighdoll, P., and Tiwari, S. N., "Infrared Radiative Heat Transfer in Nongray Gases," *International Journal of Heat and Mass Transfer*, Vol. 10, 1967, pp. 1521–1532.

[4] Greif, R. and Habib, I. S., "Infrared Radiation Transport: Exact and Approximate Results," *Journal of Heat Transfer*, Vol. 91, 1969, pp. 282–284.

[5] Mandell, D. A. and Cess, R. D., "Radiative Energy Transfer within a Hydrogen Plasma," *International Journal of Heat and Mass Transfer*, Vol. 13, 1970, pp. 1–12.

[6] Mandell, D. A., "Conduction, Convection, and Radiation in a Nonisothermal Hydrogen Plasma," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 9, 1969, pp. 1553–1561.

[7] Gilles, S. E., Cogley, A. C., and Vincenti, W. G., "A Substitute-Kernel Approximation for Radiative Transfer in a Nongray Gas Near Equilibrium, with Application to Radiative Acousics," *International Journal of Heat and Mass Transfer*, Vol. 12, 969, pp. 445–458.

[8] Mandell, D. A. and Cess, R. D., "Radiative Transport Quantities for a Hydrogen Plasma," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 9, 1969, pp. 981–994.

[9] Sparrow, E. M. and Cess, R. D., *Radiation Heat Transfer*, Brooks/Cole, Belmont, Calif., 1966, p. 227.

Integral Solution for Erosion Heat Transfer

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1. Introduction

When a heated solid is in a chemically reactive environment, under excessive pressure and shear forces, or impacted by high-speed particles, erosion may occur at the surface. The erosion phenomena have received substantial attention in recent years due to their importance in high-speed and high-temperature applications. For example, when a vehicle traverses a rain or dust environment, failure may result due to surface erosion coupled with aerodynamic heating. Erosion generally occurs over a wide temperature range and its rate is often strongly dependent on surface temperature. Erosion thus differs from the ordinary ablation process, such as sublimation, which can occur over a range of temperatures but whose rate is determined primarily by the surface heat balance. Moreover, erosion and ablation may occur simultaneously.

In the present analysis, the integral method is applied to solve the transient heat conduction problem involving an eroding surface. The general formulation considers a semi-infinite solid with temperature-dependent properties and an erosion rate that varies arbitrarily with surface temperature and time. Sample closed-form solutions are presented for the cases of erosion with convective heating and of simultaneous erosion and ablation under constant erosive environments.

2. General Formulation

The governing equation for one-dimensional heat conduction in a semi-infinite solid is

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) \tag{1}$$

where the density ρ, specific heat c, and conductivity k are considered as functions of temperature T. The location of the moving surface at time t is defined by $x = S(t)$. With the introduction of

$$z = x - S \tag{2}$$

and

$$\theta = \int_{T_\infty}^{T} \rho c dT \tag{3}$$

Equation (1) becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[\alpha(\theta)\frac{\partial \theta}{\partial z}\right] + \dot{S}\frac{\partial \theta}{\partial z} \tag{4}$$

where $\alpha = k/\rho c$, $\dot{S} \equiv dS/dt$, and the subscript ∞ refers to $x \rightarrow \infty$.

Goodman, who suggested the transformation given by Eq. (3), has applied the integral technique to solve a wide

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